LCP 14: THE PHYSICS OF STAR TREK **April 22**

Fig. 1 The Starship Enterprise

This LCP is based largely on an article published in the special Christmas edition of 1981 of the British journal *New Scientist*:

"The Physics of Star Trek" by Arthur Stinner and Ian Winchester. (See references)

In addition, the following article is used:

Stinner, A. and Metz, D. (2006). Thought Experiments, Einstein, and Physics Education. *Physics in Canada*, pp. 27-37. (Nov./Dec. 2006). (The article can be downloaded from the website of the author).

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http://home.cc.umanitoba.ca/~stinner/stinner/pdfs/2006-thoughtexperiments.pdf

"But I canna change the laws of physics, Captain!" - Scotty, to Kirk.

Captain James T. Kirk: "Evaluation, Mr. Spock".

Commander Spock: **"**It's life, Captain, but not life as we know it".

Captain Kirk is often heard saying such things as: "Let's come to a full stop", and "Let's turn around", when the Enterprise is at high speeds, say .5 c.

What problems would you encounter in carrying out these commands?

LCP 14 STAR TREK

Fig. The crew of the Spaceship Enterprise: Capt. Montgomery ,"Scotty", Scott. Commander Leonard 'Bones' McCoy, M.D., Commander Spock, and Captain Kirk.

IL *** A brief history of the original Star Trek Series
http://en.wikipedia.org/wiki/Star_Trek_TOS

We will study motion in the three regions of physics:

1. Speed of less than 10% the speed of light (Newtonian),
2. Speed greater than 10% but less than the speed of light (Einsteinian), and
3. Speed greater than the speed of light (superluminal, or tachyon-like).

Videos:

ILV 1 ***
http://video.google.ca/videosearch?hl=en&q=Star%20trek&um=1&ie=UTF-8&sa=N&tab=wv#
Look at:
Star Trek: The Last Episode. (Very entertaining)
Top 10 Star Trek Technobabbles (Science and science fiction)
ILV 2 ***
http://video.google.ca/videosearch?hl=en&q=physics%20and%20Star%20trek&um=1&ie=UTF-8&sa=N&tab=wv#
Look at:
Science of Star Trek: Transporters
The Physics of Star Trek Review

LCP 14 STAR TREK
The Physics of Star Trek Warp Drive
The Physics of Starship Firepower- Phasers & Disintregration
The Physics of Movie Monsters: The Cube Square Law
Don's Frickin' Physics Project
"Jumper" & Real Teleportation

Fig. The Enterprise in action

IL * Brief biography of Star Trek**
http://en.wikipedia.org/wiki/Star_Trek

THE MAIN IDEA

The TV series Star Trek has captured the imagination of generations of students since it was introduced in the late 1960's. The original cast was later replaced by a new one in the series "Star Trek, the New Generation". However, those who have grown up with the original series seem to prefer it, as attested by the fact that the new series is run concurrently with the old one.

The episodes of Star Trek concentrate on speculating on the personal and social interaction in a galactic society of the twenty third century. For such a society to be able to interact at all we must postulate a high technological achievement, indeed. The level of technological and scientific achievement is indicated by having the Star Ship Enterprise travel at a speed faster than that of light, and making the teleportation of the crew from the surface of a planet to the Enterprise seem plausible.

While some of these are not impossible, according to contemporary physical theory particles with a rest-mass cannot travel faster than the speed of light, in fact cannot travel as fast as the speed of light.

LCP 14 STAR TREK

Moreover, teleportation, *a la* Star Trek seems to contradict the second law of thermodynamics (see: "The Notion of Energy").

In Star Trek the dramatic setting therefore is based on two assumptions:

1. That an Earth Star Ship can make contact with other civilizations in other star systems and even galaxies,
2. That Captain Kirk, Mr Spock, Dr. McCoy, and the rest of the crew are isolated from their star base on earth so that they must make all crucial decisions affecting these civilisations by themselves. These assumptions in turn presuppose (take it for granted),
3. That faster-than - light travel for a physical object like a Star Ship be possible,
4. That extraordinary means of communication involving faster than light transmission be available to the Enterprise in communicating with Starbase.

Our strategy will be to devise a sequence of problems and invite you to consider a series of questions that would naturally occur to Captain Kirk and Mr. Spock in travelling on their journeys or planning them. Since this is an advanced assignment it will not be specifically spelled out what you must know before attempting to solve the problems. The physics required beyond elementary relativity theory, however, will be discussed in some detail. As well, the relevant equations will be provided.

Part A deals with the physics of space travel using only Newtonian mechanics. In part B we shall deal with problems that require elementary relativistic mechanics only. Part C deals speculatively with faster-than-light travel by means of plausible extensions of the special theory of relativity and the strange physics it produces. For all problems assume that the length of the Enterprise is 200 m and its rest mass 190 000 tonnes, or 1.9×10^8 kg, mass of spaceship plus fuel.

Note to the student: In the following sections we will discuss the problem of space travel. These problems are not intended to give you a deep understanding of relativity. Neither are they supposed to teach you to wield "formulas" without understanding. Rather, it is hoped that by (as Einstein put it) *playing with concepts,* the physics of motion will be a source of delight as well as mystery.

LCP 14 STAR TREK
Interstellar Cantering (The Newtonian regime)

The nearest star to us, Proxima Centauri, is about 4.3 light years away. Assume that the maximum speed of today's rockets, relative to earth, is just a little more than the escape velocity (53 km/s) needed to leave the solar system from Earth. The distance to the sun is about 1.5×10^{11}m. Asume that the acceleration and deceleration should be about 1g or 10 m/s^2.

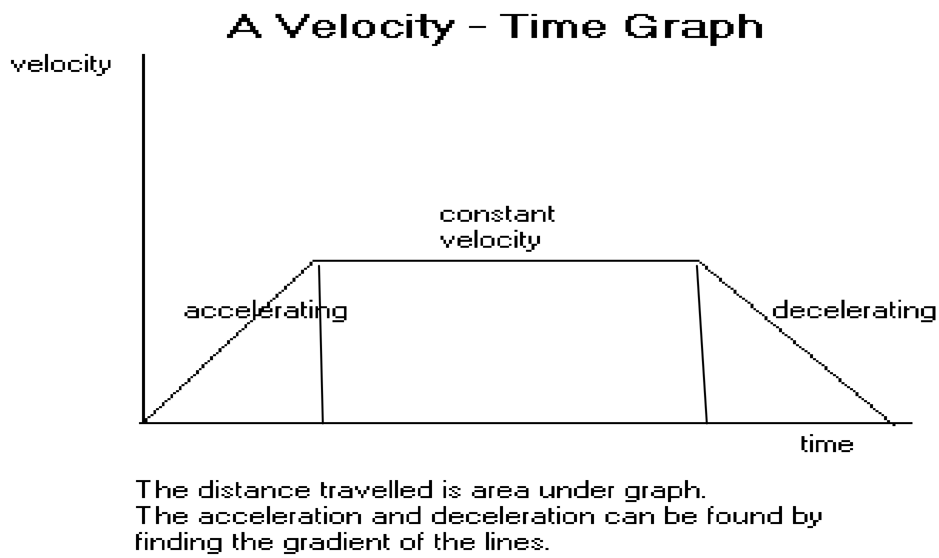


Fig. The general velocity time graph for travelling in space

Note that :

1. The slope of the graph is g (about 10m/ s^2) and negative g.
2. The times for accelerating and decelerating are negligible for long distances (distances between stars, for example).

LCP 14 STAR TREK

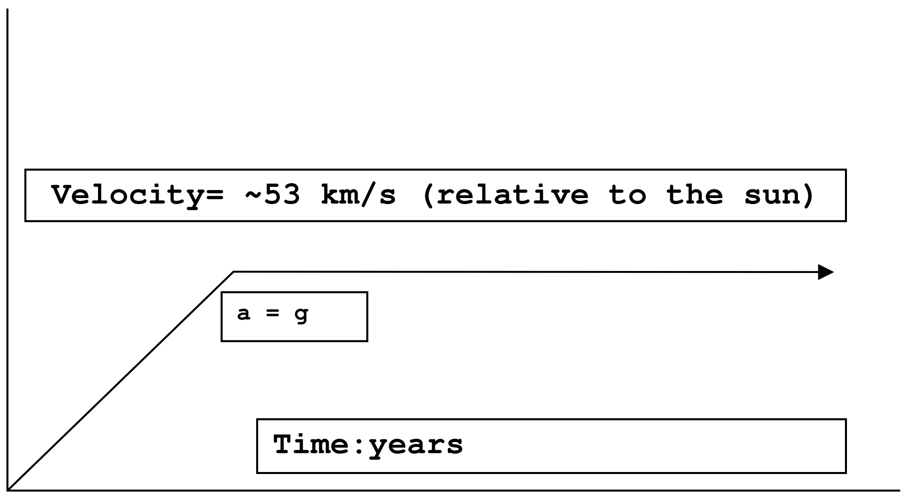


Fig. A trip to the planets beyond Jupiter would look like this. The time to accelerate would be short compared to the time of flight. The velocity of the rocket (relative to the Sun would have to be about 53 km/s.

In Fig. we have velocity time graph of a SC (rocket) travelling into space. The slope of the graph is the acceleration and should be about 10 m/s^2, the acceleration due to gravity on Earth that we are used to. The total distance is the area under the graph.

However, we must remember that Newtonian physics is only good for velocities of up to about 0.1 c. See the velocity time graph below (actually an energy-time graph). We will begin by making a few calculations based on today's technology to show that interstellar travelling is not possible within the lifetime of humans.

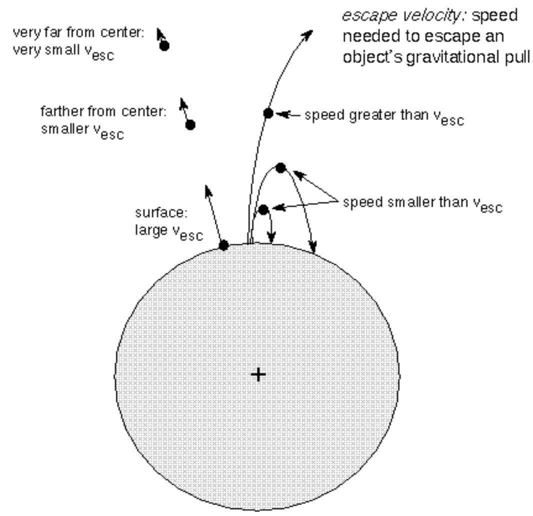
<u>Discussion:</u>

The escape velocity from the solar system, starting from the Earth's surface is very high, about 53 km/s. As we have already shown in LCP 2, the escape velocity from Earth is about 11.2 km/s. The Earth moves with a speed of about 30 km/s around the Sun. In addition, the escape velocity from a circular orbit is $\sqrt{2}$ times the orbital velocity of a satellite. (See LCP 2). This adds up to about 53 km/s.

LCP 14 STAR TREK

To achieve escape velocity using a rocket of total mass $\mathbf{m_i}$ (total mass: mass of rocket plus mass of fuel), we can only have a "payload" of less than 1 %. You can show this by using the rocket equation we developed in LCP2 and in LCP 5 II: $\mathbf{v = v_e \ln m_i / m_f}$

Assuming an escape velocity of the gas that propels the rocket to be about 5 km/s you can show that only about less than 1% of the total mass of the rocket can be a payload, that is, for every kg of payload you would have to burn about 40,000 kg of fuel.!



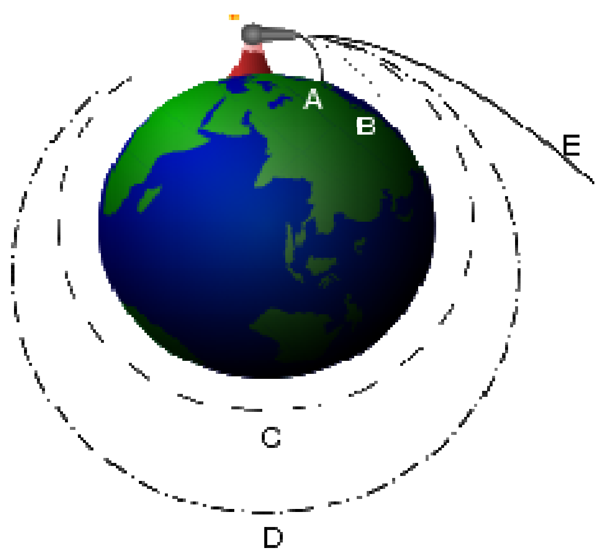


Fig. Isaac Newton's analysis of orbital motion and escape velocity. Projectiles A and B fall back to earth. Projectile C achieves a circular orbit, D an elliptical one. Projectile E escapes.

LCP 14 STAR TREK

(I) Elementary equations for space travel, using Newtonian dynamics

Rocket equation:

$$v = v_e \ln (m_i/m_f),$$

Kinetic energy equation:

$$E = \tfrac{1}{2} mv^2$$

where $\mathbf{m_i}$ is the initial mass (mass of rocket plus mass of fuel) and $\mathbf{m_f}$ is the final mass of the rocket (payload). See LCP 2 and LCP 5II..

Travelling in the solar system

IL ** Travelling in the solar system**

http://www.classzone.com/books/earth_science/terc/content/visualizations/es2701/es2701page01.cfm

A wonderful animation to give a sense of the vast distance involved to travel to the outer regions of our solar system. Taken from IL :

At the speed of today's fastest spacecraft (~20 km/second), it would take almost ten years to travel this distance. Even at the speed of light, the trip would last 5 1/2 hours. In this animation, the apparent speed of the viewer is over 300 times the speed of light.

Questions and problems

1. How long would it take a rocket to travel to Neptune, a distance of about 30 AU? The speed you are allowed is the escape velocity from the solar system (starting from Earth) or about 53 km/s.
2. How long would it take to reach the nearest star, about 4 LY (light years)? Why is such an undertaking, even if it were achievable, not realistic?
3. The energy required for such a trip is large but not beyond today's technological capabilities. Refer to the Table I and calculate the energy involved and the mass of propellant required for such a trip. Clearly, the energy is expended only during the acceleration and deceleration periods and very little during the near constant speed trip.

   That means that the energy required to travel to the nearest star would be almost the same. This may be a surprise to most students. (Remember that the gravitational force falls off inversely as the square of the distance.)

LCP 14 STAR TREK

4. Discuss the feasibility of such a trip with respect to (i) the time duration, and (ii) the energy expenditure.
5. Imagine that you had the energy available to accelerate at 1 g (to produce an artificial gravity like that of an RRS to a maximum speed and then decelerate at 1 g to the nearest star, 4.3 light years away. Remember you are using Newtonian physics,
   a. What would be the maximum energy of the rocket?
   b. How long would the trip take, as measured from the Earth and as measured in the rocket?
   c. How much energy per kg mass would it require?
   d. Compare this energy with the energy available in Table I. Comment

For the following see LCP 13. In addition, the formulas will be discussed in more detail later in this presentation

(II) Relativistic kinematics: (See LCP 13)

IL **** An excellent visual description of the STR.

http://abyss.uoregon.edu/~js/ast122/lectures/lec20.html

1. $\mathbf{t = t' / (1-v^2/c^2)^{1/2}}$ (time dilation)

where t is the time *as measured on Starbase*, t' is the time as *measured by the crew on spaceship*, and c is the velocity of light in vacuum: $3x10^9$ m/s.

2. $\mathbf{l = l' (1-v^2/c^2)^{1/2}}$ (length contraction)

where l is the distance travelled *as measured by the crew on spaceship* and l' is the distance *as measured on Starbase*.

(III) Relativistic dynamics

3. $\mathbf{E = m_0c^2 / (1-v^2/c^2)^{1/2}}$

where $\mathbf{m_0}$ is the "proper mass" (mass as determined in the spaceship)

4. $\mathbf{p = m_0 v / ((1-v^2/c^2)^{1/2}}$ (For relativistic momentum equation)

where **p** is the relativistic momentum and **v** is the velocity of the rocket.

LCP 14 STAR TREK

(IV) Relativistic addition of velocities

$$\mathbf{V_{relative} = [(v' + v) / (1 + v' v / c^2)\ ]}\quad \text{(absolute value!)}$$

where $\mathbf{V_{relative}}$ is the relative velocity , or the velocity as measured from Starbase.

Note that velocities are vector quantities and therefore the absolute value is taken for $\mathbf{V_{relative}}$.

(V) Relativistic kinematics of constant acceleration

Comment for V: The following is taken from ( ):

It might be argues that, since the STR compares only inertial frames of reference ewe cannot deal with the problem of acceleration. The following explanation for how this can be done is taken fro IL :

*First of all we need to be clear what we mean by continuous acceleration at 1*g. *The acceleration of the rocket must be measured at any given instant in a non-accelerating frame of reference travelling at the same instantaneous speed as the rocket. This acceleration will be denoted by* a. *The proper time as measured by the crew of the rocket will be denoted by* T *and the time as measured in a the non-accelerating frame of reference in which they started will be denoted by* t. *We assume that the stars are essentially at rest in this frame. . The distance covered as measured in this frame of reference will be denoted by* d *and the speed* v. *The time dilation or length contraction factor at any instant is a function* of *gamma.*

We will derive some of these equations later.

1. $\mathbf{v = at / \{(1 + (at/c)^2\ \}^{1/2}}$

where **a** is the acceleration of the rocket as *determined in the spaceship by the force of a unit mass on a spring*; **t** is *the time as measured from Starbase*, and v is *the velocity as measured from Starbase.*

$$\mathbf{2.\quad t' = c/a\ arcsinh\ (at/c)}$$

where **t, t'**, and **c** have been defined before, and

$$\mathbf{3.\quad L = c^2/a\ \{[\ 1 + (at/c)^2]^{1/2} - 1\}}$$

where **L** is the distance travelled *as measured from Starbase*, and **a** and **c** have already been defined.

$$\mathbf{4.\ T = \{ 1 + (at/c)^2 \}^{1/2}}$$

where **T** is the time duration as measured in the space craft.

How good is Newtonian physics for high speeds?

1. If you applied Newtonian mechanics to study the motion of the Enterprise at "low" speeds (speeds less than 0.1 c) you would find that the equations would give you reliable answers. However, at high speeds these equations are not valid any more. Test this statement for velocities of 0.01 c, 0.1 c, and 0.99 c. Comment.
2. When the velocity of the Enterprise is 0.1 c what percentage error will you have in your calculation of energy, momentum, etc., based on Newtonian mechanics?
3. At what velocities will the error of the calculations be more than 100%?
4. When the Enterprise is travelling with v = .99 c, what is its mass, length, and kinetic energy, as measured from the inertial frames of reference of 1. Starbase, and 2. the Enterprise

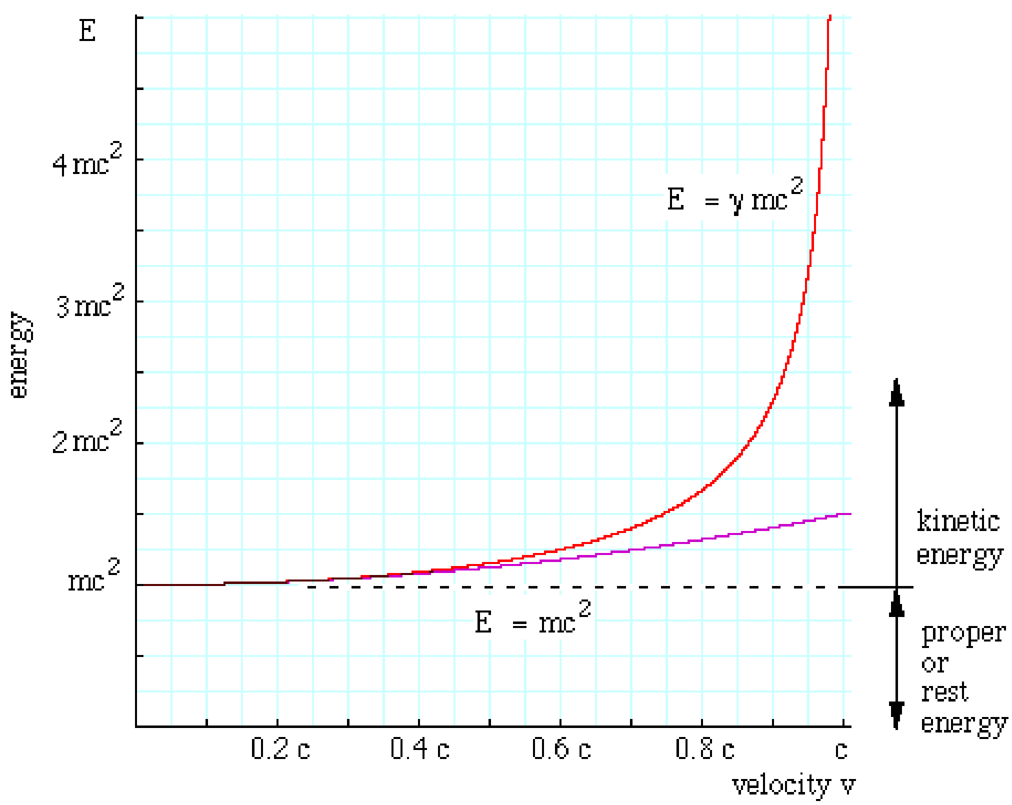


Fig. Velocity-time graph for relativistic motion. Purple line is Newtonian and red line is Einsteinian.

LCP 14 STAR TREK

Research problems for the student:

1. Because mass is ejected during the acceleration and deceleration phases only, the actual energy required is not given by the expression ½ Mv^2. Discuss first and then look up the solution to the "general rocket problem" in one of the college texts cited in the references.

Fig. The Enterprise in deep space

2. Captain Kirk orders a leisurely voyage of 0.1 c velocity from earth to Proxima Centauri (c is the speed of light). The acceleration and deceleration phases are to be of equal length at 1 g (10 m/s^2). The selection of an acceleration of 1 g would, of course, provide an artificial gravity that is earth-like. Consider the acceleration phase "Newtonian" for both cases.
3. How long would the trip take, assuming Newtonian physics still worked at this speed, and assuming the energy is available?
4. In many episodes of Startrek one hears Scottie say: "use rocket propulsion". If the rockets of the Enterprise were using conventional fuel (H_2 O_2, for example) and the exhaust velocity (relative to the starship) is 4 km/s, what would be the value of M_i/M_o for the rocket, for v = 0.1 c, using the rocket equation given in t Appendix 1?
5. If Scottie ordered the use of ion thrust with an exhaust velocity of 100 km/s, what would be the value of M_i/M_o now?

. The above sequence of questions should make you realize the limits imposed by the two physiological constraints: accelerations cannot be much higher than 1 g for sustained acceleration periods and that gravity-free coasting time of more than a few months tends to produce a human jellyfish. The M_i/M_o ratio calculated for conventional and ion thrust propulsion should show you the impracticality of reaching high speeds with other than matter-antimatter annihilation propulsion. Discuss briefly by elaborating on these points.

3. Approaching a solar system in quadrant IV A, sector 13, Mr. Spock reports that sensors indicate a class m (Earth-like) planet. Captain Kirk gives orders to manoeuvre the Enterprise into a circular orbit around the planet. Before this can be done, however, the mass of the planet must be determined. In order to do this Mr. Spock uses the information that a very small moon, with a mass of approximately 10^{11} kg, is in a circular orbit with a period of 90 minutes, approximately 500 km above the planet's surface. The diameter of the planet is optically determined to be about 1.3×10^{7}m.

As soon as the Enterprise is in orbit Captain Kirk orders a landing party to be beamed down. He turns to Spock and says: "By the way, Spock, what is the gravity on the surface?" Raising his right eyebrow Spock answers: "Earth_like, sir." Confidently the party enters the transformer room and beams down to the surface. Calculate the approximate value of the surface gravity. Was Spock right?

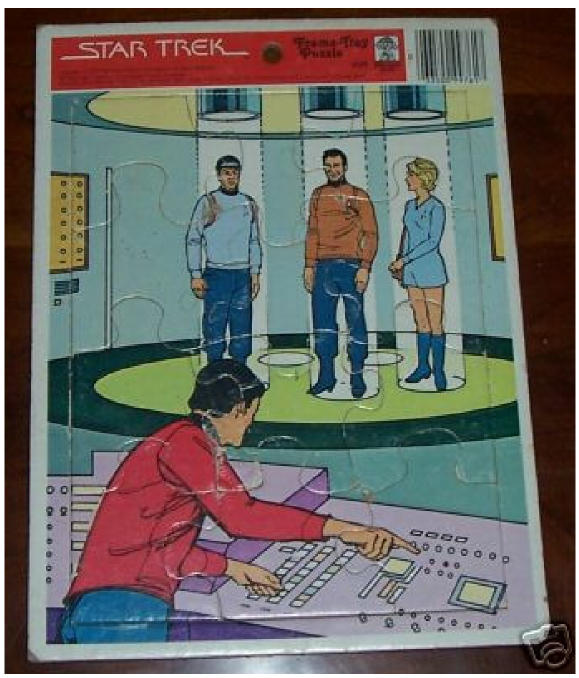


Fig. Teleportation in Star Trek

Interstellar Trotting

The successful solution of the problems in part A should have convinced you that interstellar journeys in a single lifetime at leisurely speeds of 0.1 c or less are simply not possible, given the constraints of time and energy requirements. In this section we shall see that for speeds between 0.1 c

LCP 14 STAR TREK

and c the special theory of relativity enables the prediction of at least the possibility of interstellar space travel.

We shall, however, discover that in order to make such journeys the theory predicts the possibility of space travel compatible with Star Trek requirements, but incompatible with energy requirements. Moreover, for Star Trek purposes one would like to engage in interstellar travel and return to Starbase five years later essentially unchanged physiologically relative to Starbase time. However, the *twin paradox* principle would seem to violate this dramatic requirement. The following sample questions explore these problems of elementary relativistic space travel.

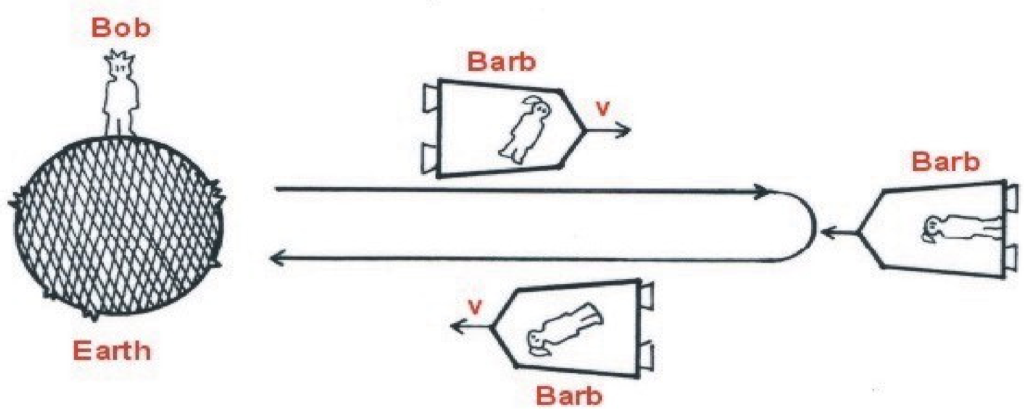


Fig. The Twin Paradox. See LCP 13

Relativistic space travel (See LCP 13)

1. The quantity $\mathbf{R = (1\text{-}v^2/c^2)^{1/2}}$ is fundamental in transforming the equations of Newtonian mechanics to those of relativistic physics. You can see from Table II that this quantity occurs as either R or as its reciprocal 1/R.
   a. As a preliminary exercise plot the quantity 1/R against v and then relate this graph to energy, mass and momentum. This will tell you how these important physical quantities, as measured from Starbase, change with the velocity of the Star Ship.
   b. Now plot R against v and relate this graph to the time and distance travelled. For example, this graph will tell you how the distance and time, as measured from the frame of reference of the starship, changes with the velocity of the Enterprise.

Reaching the near-by stars

LCP 14 STAR TREK

1. Captain Kirk is in a hurry to reach Proxima Centauri. Assume that the Enterprise is able to accelerate and decelerate to very high velocities in short time intervals and that the crew is "protected" against the law of inertia.

   a. How long would it take (ship time) to reach the star if the velocity is .99998c? What time interval would that correspond to on Starbase?

   b. Our present understanding of physics and physiology would prohibit accelerations of much over one g (10 m/s^2). Microbes, on the other hand, can be accelerated in super centrifuges to about 10 000 g's without being destroyed. The Star Ship Enterprise seems to be capable of accelerations of that order with no apparent ill-effects on the crew.

On the other hand, we know that when we jump from a diving board, for example, we feel no force. Does that mean that if we fell freely in any gravitational field we would feel no force?

Perhaps, the Enterprise has the ability to "generate" an appropriate gravitational field for *force-free* acceleration? Discuss.

The energy requirements for long journeys in a galaxy

Fuel	Types	Final Product	Energy available x 10^{14} (J/kg) Mass
$H_2 - O_2$	**Chemical**	**H_2O**	**$6.3\ 10^{-8}$**
Uranium	**Fission**	**Iron**	**1.1**
H-H	**Fusion**	**He**	**6.3**
Matter-Antimatter	**Radiation**	**Photons**	**900**

Table I: Energy Availability

Starbase decides that an extraordinary journey to the edge of our galaxy is Captain Kirk's next mission. The distance is 10 000 light years. Assume that at sub-light speeds the Enterprise is capable of reaching 0.9999999998 c and that acceleration and deceleration times are negligibly small.

a. Calculate the mass of matter-antimatter that would be required to travel to the edge of the galaxy.

b. How long would the journey take, measured on the Enterprise? On Starbase?

Note: This problem demonstrates that even if 100% matter-antimatter energy source is assumed (the ultimate energy source, so far as we know) most of the mass of the Enterprise would have to consist of fuel

LCP 14 STAR TREK

Discussion in preparation for solving the problems below...

1. Rapid accelerations and decelerations have required Dr. McCoy to work overtime with cell-repair surgery, so Captain Kirk orders a return trip in which the Enterprise is to accelerate and decelerate uniformly to and from the midpoint of the journey, X light years away, when a = g or 10 m/s^2 and maximum velocity is not to exceed 0.9999999998 c.

   a. Calculate the maximum distance the Enterprise could travel with these constraints.Assume that sufficient energy is available for the trip.

   b. Now calculate the maximum velocity reached and the times of the trip measured by Starbase and by the crew of the Enterprise respectively, for a distance of 100 light years, 1000 light years.

   c. You may have suspected by now that the energy available is not sufficient for this kind of trip. Calculate the ratio of M_i/M_o.

   d. What then is the maximum distance the Enterprise could travel given that only 90% of its mass can be used as fuel for the matter-antimatter energy source?

2. In the light of what you have discovered about space travel, especially the energy requirements for such travel, discuss the general question of the possibility of interstellar travel.

3. The Star Ship Enterprise often travels vast distances at speeds close to the speed of light. The mandate of the crew was to explore space for a five year period (as measured by clocks on board). Thus the crew would have aged five years when they returned to Starbase. What and whom would they find on returning to Starbase, according to the special theory of relativity? Discuss.

LCP 14 STAR TREK

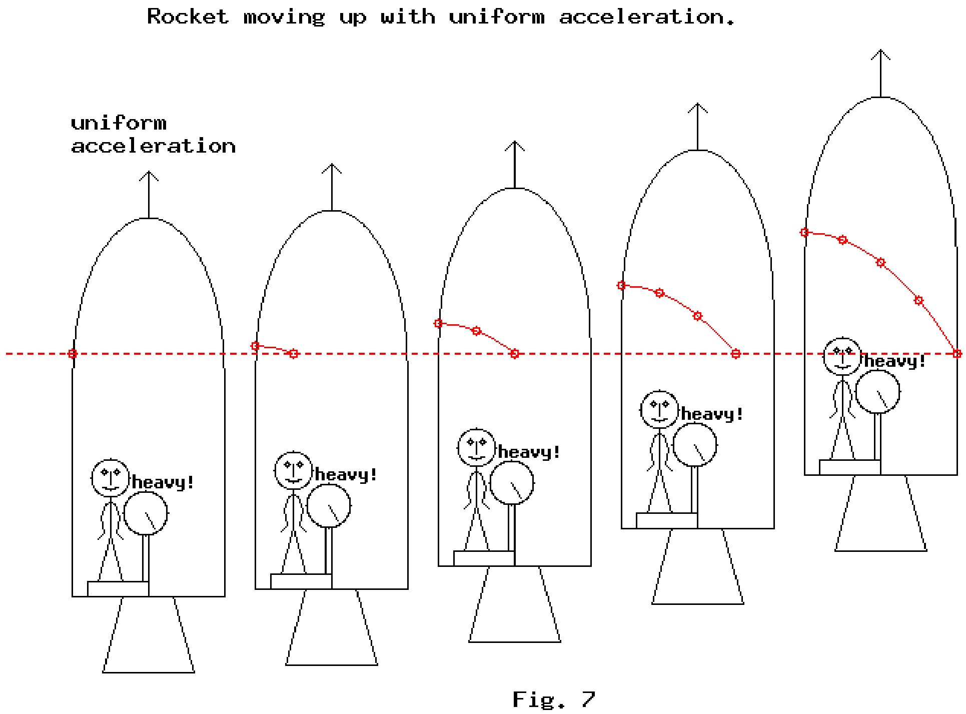


Fig. Rocket moving in deep space: Einstein's equivalence principle.

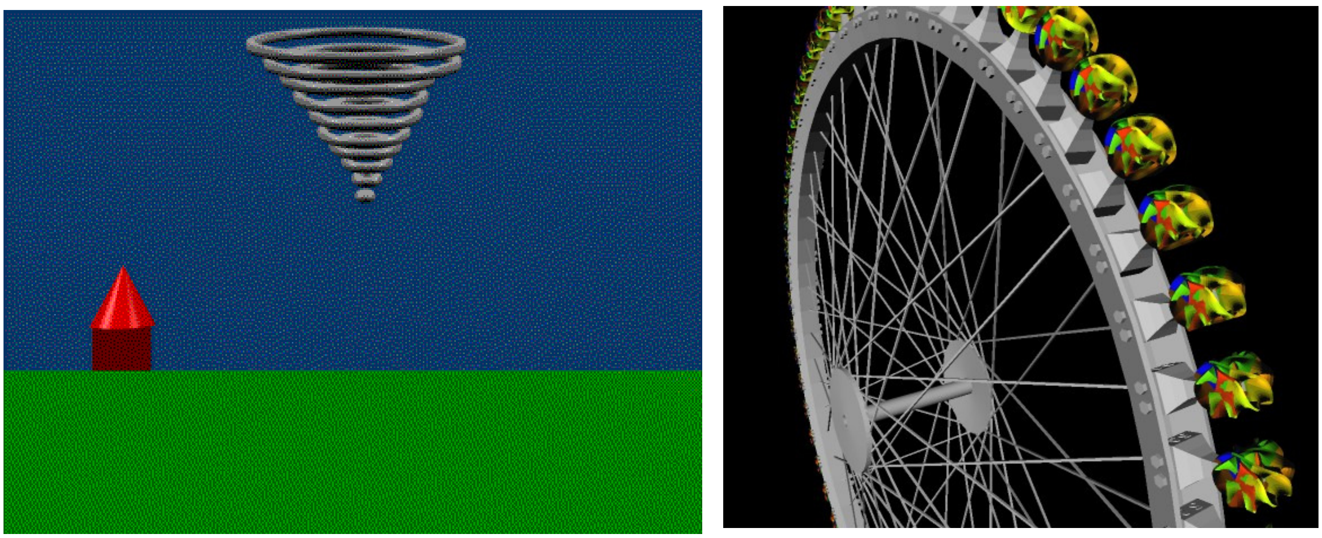
8.5 Intergalactic Galloping

From sample problems and questions as those in Parts A and B it is clear that at sub-light speeds the Enterprise would never be able to travel between stars, not to speak of intergallactic in the lifetime of its crew. Moreover, it is clear that for any conventional fuel the quantity to be carried would be vastly more than the mass of the Star Ship itself. Indeed, as we have seen, even if we assumed that the source of the rocket drive was the direct conversion of matter into energy the result would be similar. Thus if we were to have slower than light travel between stars and galaxies that did not involve multiple lifetimes and intolerable quantities of fuel on board, we would require both longer human lifetimes and the ability to somehow extract energy from space *en route*. For such reasons as these, faster-than-light travel, were it physically and humanly possible, would be very attractive indeed.

Before Albert Einstein's 1905 article on the electrodynamics of moving bodies, in which he raised the hypothesis that $c = 3x10^8$ m/s was a universal constant to the status of a postulate, intuition seemed to be against the notion of a "fastest speed". Gottfried Leibniz, Isaac Newton's great contemporary, had an argument to show that this notion was in fact self-contradictory. The argument went roughly as follows.

LCP 14 STAR TREK

Assume that one has a spoke wheel every point on the rim of which is rotating at the fastest speed. Then we can always extend the spoke an arbitrary distance beyond the rim. But any point on the extension of that spoke must be covering a greater distance than any point on the rim during the revolution of the wheel. Consequently the point on the rim cannot be travelling at the fastest speed, contrary to our initial hypothesis.



Fig, The thought experiment of Leibniz

Of course, Leibniz assumed that the objects at any speed would retain their rigidity a *dimensional permanence*, that is, their dimensions as measured from any6 frame of reference would be the same.
As we have seen, Einstein's STR changed this assumption.

A number of contemporary physicists, including Gerald Feinberg, Yakov P. Terletkii, Olexa-Myron Bilaniak and George Sudarshan have argued that faster-than-light travel for particles is in no way precluded by Einstein's special theory of relativity. There is, as yet, no experimental evidence that faster-than-light particles exist. But this is not surprising since their properties would be such that their detection would be very difficult.

Such a situation offers us a unique opportunity to try to think and calculate well at an elementary level about matters at the limit of our physical understanding. In the context of a programme like Star Trek we already have thought about these matters and the implications involved. (You are now urged to read the article cited in the references by Gerald Feinberg.)

Our predictions of the properties of faster-than-light particles, or *tachyons*, are in the main derived from relativistic kinematics. Let us begin this exciting topic with the following questions.

LCP 14 STAR TREK

Hyper-drive: Faster-than-light-travel

1. When Mr. Spock, as science officer aboard the Enterprise, wishes to make some quick calculations about some upcoming faster-than-light travel he contemplates equation III in Table II, namely:

$$\mathbf{E^2 = M^2\, c^4 / (1\text{-}v^2/c^2)^{1/2}}$$

for the case in which v is greater than c.

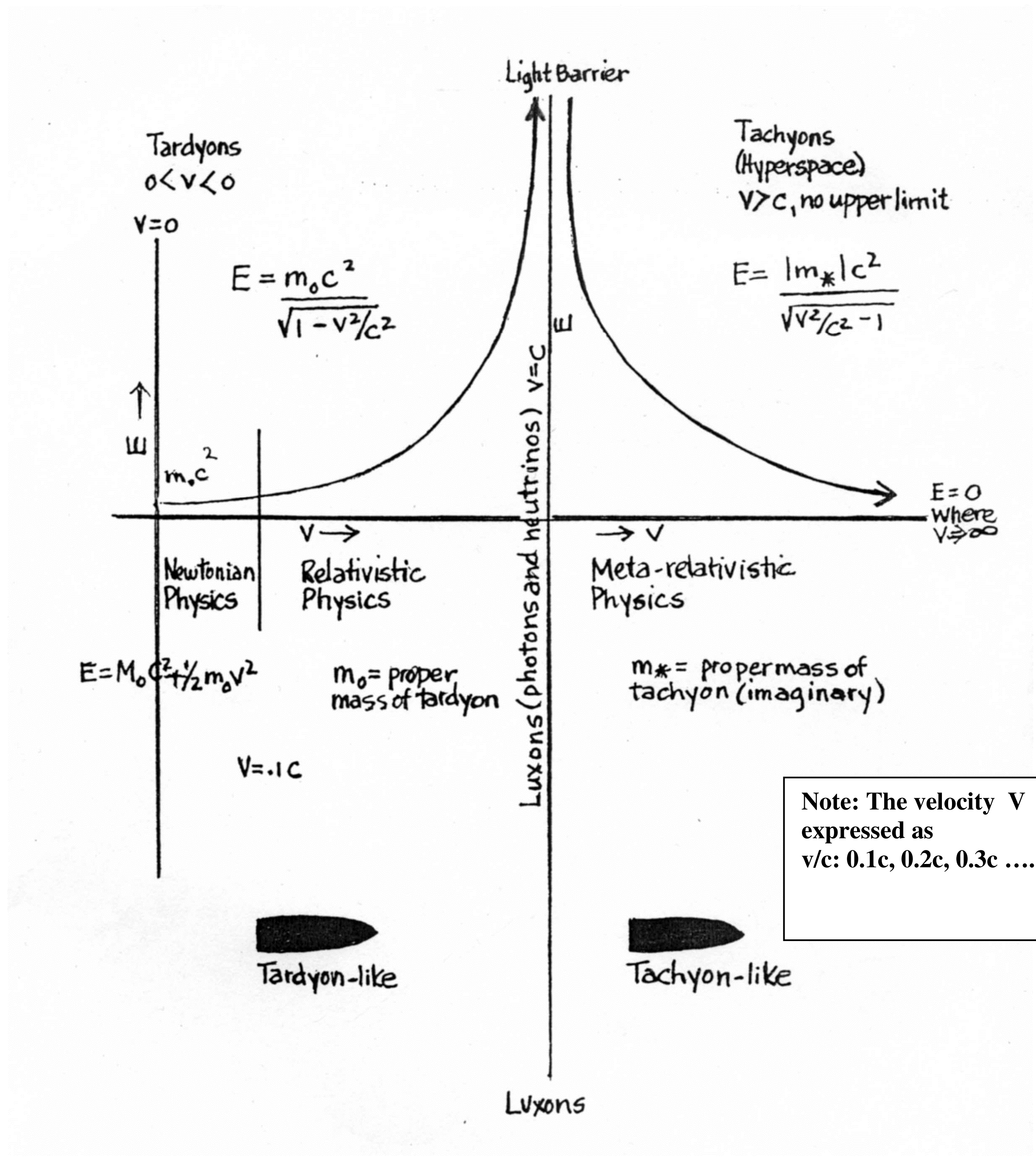
a. He usually writes this equation as

$$\mathbf{E^2 = M^2\, c^4\, (1\text{-}v^2/c^2)}$$

where E^2 is assumed to be an intrinsically positive number. For this case, namely for positive E^2 and for v greater than c, what must be the mathematical character of the quantity M^* ? Is an imaginary mass any more puzzling as a physical quantity that there are *luxons* (photons and neutrinos) which are particles that travel only at the speed of light and which, though they have a relativistic mass given by $m = E/c^2$ nonetheless have a zero rest mass? Discuss.

b. Spock always has a graph of $\mathbf{E} = M^*\mathbf{c^2 / (v^2/c^2\text{-}1)^{1/2}}$ plotted as well as $\mathbf{E = Mc^2 / (1\text{-}v^2/c^2)}$**,** so that he can rapidly estimate energy-velocity relationships.

LCP 14 STAR TREK



Note: The velocity V expressed as v/c: 0.1c, 0.2c, 0.3c ….

Fig. Commander Spock's "Travel Guide". (Note: v is expressed as as a fraction of c)

LCP 14 STAR TREK

2. In order to escape a Klingon warship chasing him Captain Kirk orders the Enterprise to go into *hyperdrive* (that is, into faster-than-light travel).

   a. When the Enterprise crosses the speed of light barrier, need we assume that its mass undergoes any intrinsic change? Should we assume that for observers on board ship no difference in nature of its matter could be noticed? Would there be a difference from the vantage point of Starbase?

   b. When thinking about faster-than-light travel Spock has to keep in mind some differences which his graphs suggest to him. For example, for slower-than-light travel acceleration of particles requires energy. For particles travelling at the speed of light (the luxons) the notions of acceleration and deceleration do not apply. Why not? Discuss.

   c. Spock also knows that *tachyons* (faster-than-light particles) naturally accelerate. Why is that? Should it take energy to decelerate a tachyon or to keep it moving at a constant speed? Or is there any sense to such a notion? Discuss.

   (Note: Clearly, for the Starship Enterprise it is important that when it is in hyperdrive it can be kept at a constant faster-than-light speed or in a decelerated state.)

Discussion in preparation for solving the problems below...

1. After pursuing a Klingon spaceship with an offer of friendship Captain Kirk abandons the attempt when it becomes clear that the pursuit would take him to the edge of the Galaxy. He then gives order to "turn around" and proceed in the opposite direction back to Starbase. The pursuit begins at low speeds and proceeds close to that of light. Then the Enterprise is put into *hyperdrive,* reaching high speeds, just before the decision is made to travel back to Starbase.

   Calculate the relative speeds between the two spaceships, relative to (as "seen" from star base) when:

   (i) The speed of the Enterprise is 0.04 c and that of the Klingon spaceship is 0.88 c, when both are travelling in the same direction and when they are travelling in opposite directions.

   (ii) The speed of the Enterprise is 0.98 c and that of the Klingon spaceship

0.88 c. Compare the relative speeds for the cases above with the answers you would obtain if Newtonian physics applied at high speeds.

(iii) The speed of the Enterprise is 0.99 c and that of the Klingon spaceship 1.3 c.

(iv) The speed of the Enterprise is 3 c and that of the Klingon spaceship 10c.

2. Captain Kirk is often heard saying such things as: "Let's come to full stop", and "Let's turn around", when the Enterprise is at high speeds, say 3 c. What problems would you encounter in carrying out these commands? Discuss.

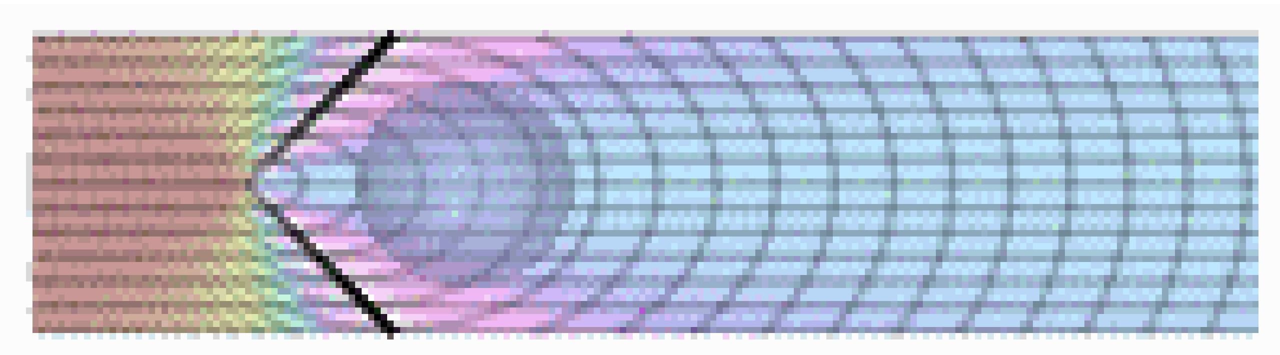


Fig. An animation showing the motion of a tachyon.

Tachyon visualization, analogous to the sound made by a supersonic jet. Since a tachyon moves faster than the speed of light, we can not see it approaching. After a **tachyon** has passed nearby, we would be able to see two images of it, appearing and departing in opposite directions.

The black line is the shock wave of Cherenkov radiation (analogous to a sonic boom), shown only in one moment of time. This double image effect is most dramatically illustrated for an observer located directly in the path of a faster-than-light object (in this example a sphere, shown in grey).

The right hand bluish shape is the image formed by the blue-doppler shifted light arriving at the observer — who is located at the apex of the black Cherenkov lines — from the faster-than-light sphere as it *approaches*; it moves "backwards" as light arrives from earlier and earlier positions of the sphere before it arrived at the observer. The left-hand reddish image is formed from redshifted light that leaves the sphere *after* it passes the observer; it moves "forward" following the sphere.

LCP 14 STAR TREK

Since the object arrives before the light the observer sees nothing until the sphere starts to pass the observer, after which the image-as-seen-by-the-observer slowly splits into two — one of the arriving sphere (to the right) and one of the departing sphere (to the left).

The problem with *tachyons*

1. Let us look at some of the puzzling results of faster-than-light physics. Some of them have been satisfactorily explained but others are open to speculation. Think about them, read some of the papers cited in the references, and attempt to offer some plausible explanations of your own. Here they are:
    a. A faster-than-light spaceship would have to lose energy in order to speed up and gain energy in order to slow down. The lower limit of speed is the velocity of light, while there is no upper limit.
    b. If the Star Ship Enterprise were moving with infinite speed as seen by one observer, its speed as seen by another observer in motion with respect to the first would not be infinite but some finite value greater than c.
    c. The number of spaceships observed in a region of space depends on the observer. Think of what implications this would have in an encounter between Klingon and Federation spaceships.
    d. The relative speed between two spaceships that travel faster than the speed of light cannot equal or exceed the speed of light.
    e. The method of communication at faster-than-light speeds is problematic and at the moment open to speculation. Can the crew now communicate? What would it be like to shoot tachyons from a Star Ship in order to communicate?
    f. The special theory of relativity restricts signals from moving faster than the speed of light. The existence of signals faster than the speed of light implies the existence of signals into the past. But this in turn suggests the possibility that cause can follow its effect or an effect can precede its cause.

LCP 14 STAR TREK

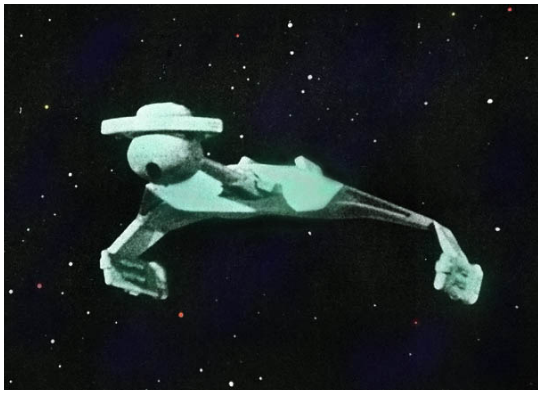


Fig. An encounter with a Klingon spaceship.

One of the big problems that the navigator of the Enterprise obviously has solved is connected with the nature of space at relativistic velocities and at velocities faster than the speed of light. The four-dimensional equations of relativistic kinematics (three space dimensions and one time dimension) suggests that the navigator of the Enterprise would have the difficult task of continually readjusting changing positions of the stars.

For example, if Captain Kirk gave orders to travel toward the North Star at a velocity of 0.9 c, the constellations Leo, Hercules, Cassioeia, Pegasus, Orion, and the Big Dipper (all these constellations ordinarily surround Polaris) would all tend to crowd together at Polaris; alas, constellations no more.

LCP 14 STAR TREK

Further, the view from the rear of the Enterprise, which at low speeds would have included the Southern Cross, Sirius, and Canopus, at speed of 0.9 c would show very few stars, not including the ones just mentioned. In fact, the Southern Cross and Canopus would, as if by magic, appear in the front view, around Polaris. Indeed, as the Enterprise approaches the speed of light, all stars would appear to converge on it. What would the crew see and how would they navigate on crossing the light barrier and entering the world of tachyon-like travel?

Questions and speculations like these involve only reasoning about simple relativistic equations. But the conclusions to which one is driven are certainly startling. If there are tachyons, as the equations relating velocity and energy we looked at suggest, such entities would have to have imaginary mass. They would have to have a limiting minimum velocity c. It would require energy to slow down these entities and as they lost energy they would accelerate! For faster-than-light particles, there must be at least one frame of reference in which the particle would be judged to have infinite velocity! Can this be derived from the energy-velocity equation alone?

We are thus led to entirely different kinds of questions. For example, by what conceivable mechanism could an entire Star Ship have its mass transformed from ordinary to imaginary and back again? The greatest nineteenth century physicist, James Clerk Maxwell, was reported to have been amazed by the transmission and detection of electromagnetic waves as developed by Alexander Graham Bell. Maxwell's amazement is noteworthy because it was he who discovered the equations that laid the groundwork for all our present knowledge of electromagnetic theory. His equations of electromagnetic theory left the discovery of the telephone as a logical possibility but for which no mechanism was suggested. Should Einstein by amazed by Mr. Spock?

Front view when travelling at various speeds from “rest” to 0.9999c

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Taken from

IL www.fourmilab.ch/cship/aberration.html Remove frame

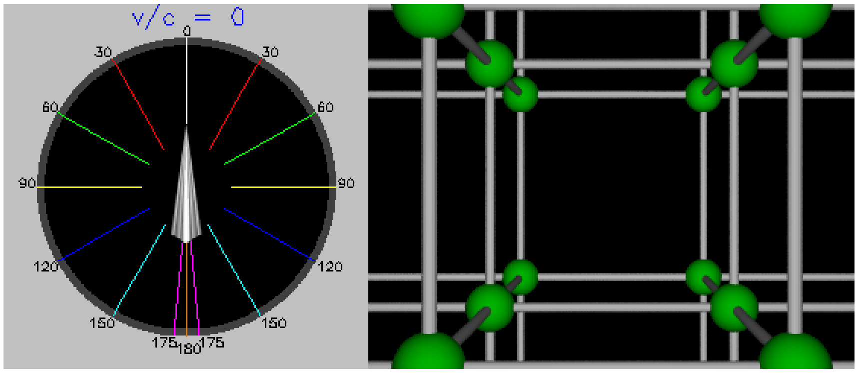

Fig. 1 At rest in the middle of the Lattice, we see a normal view, unaffected by aberration. The ship's computer displays a graphic to the left of the viewscreen that shows the effect of aberration on light arriving from various directions with respect to the direction we're travelling. Stationary in the Lattice, no aberration is indicated.

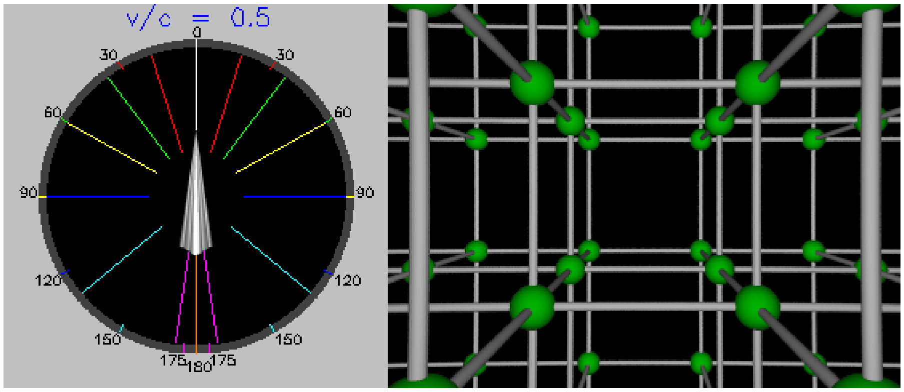

Fig. Half the speed of light, and we're developing eyes in the back of our head--objects 120° from the direction we're moving are shifted so they appear to our right and left. Still, relativity accounts for only about 10% of the total aberration.

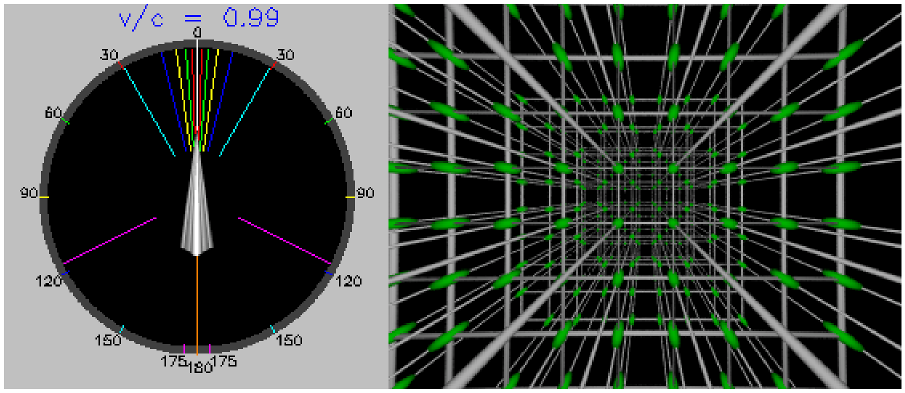

Fig. Ninety-nine percent: only objects almost directly astern still appear to be behind us.

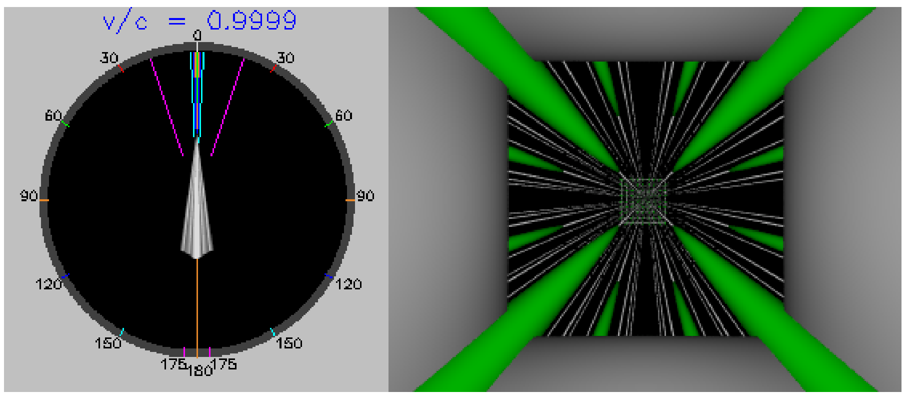

Fig. And finally, we view the Lattice from the midpoint at a velocity of 0.9999 of the speed of light. The cell of the Lattice directly behind our ship has been distorted by aberration to almost fill the field of view. The rest of the Lattice is reduced to a small grid in the centre of the main viewer. The readout to the left of the view shows that even light rays emitted five degrees from our stern appear at an angle to the bow of 15°.

APPENDIX 1:

LCP 14 STAR TREK

The following formulas are used by Spock:

$$E^2 = M^2 c^4 / (1-v^2/c^2)^{1/2}$$

$$V = (v_1 + v_2)/(1 + v_1 v_2 / c^2)$$

$$p = m\,v/[\,(1 - (v/c)^2\,]^{\cdot 1/2} = mv\gamma$$

$$(\gamma = (1-v^2/c^2)^{-1/2})$$

You can show that for small velocities (velocities less than about .05c) the expression reduces to mv. In addition. The distance **Δs** is expressed through Pythagoras' theorem as:

$$(\Delta s)^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2$$

where the symbol describes the difference in the x, y and z coordinates.

Therefore, <u>no matter who measures and determines the length, it stays the same.</u>

In relativity, however, what is invariant is not the distance but the <u>spaceBtime interval</u> given by an expression that still based on Pythagoras= theorem but includes time:

$$(\Delta s)^2 = c^2 (\Delta t)^2 - [(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]$$

The invariance of the spaceBtime interval suggests that something absolute and unchangeable underlies the physics of special relativity C a four dimensional framework that links space and time.

Time intervals and space intervals between two events depends on the observer but the space-time interval is a four-dimensional distance that takes into account all four coordinates is the same for all observers!

LCP 14 STAR TREK

Formulas required for solving the problems

(I) Elementary equations of Newtonian dynamics

Rocket equation:

$$v = v_e \ln (m_i/m_f),$$

where $\mathbf{m_i}$ is the initial mass (mass of rocket plus mass of fuel) and $\mathbf{m_f}$ is the final mass of the rocket (payload). See LCP 2.

(II) Relativistic kinematics:

1. $t = t' / (1-v^2/c^2)^{1/2}$ (time dilation)

   where t is the time *as measured on starbase*, t' is the time as *measured by the crew on spaceship*, and c is the velocity of light in vacuum: 3×10^9 m/s.

2. $l = l' (1-v^2/c^2)^{1/2}$ (length contraction)

   where l is the distance travelled *as measured by the crew on spaceship* and l' is the distance *as measured from starbase*.

(III) Relativistic dynamics

$$E = m_0 c^2 / (1-v^2/c^2)^{1/2}$$

where $\mathbf{m_0}$ is the "proper mass" (mass as determined in the spaceship)

(IV) Relativistic rocketing

1. $v = \{(1 - M^{-2}) / (1 + M^{-2})\} c$ (For matter-antimatter propulsion)

2. $p = m_0 v / ((1-v^2/c^2)^{1/2}$ (For relativistic momentum equation)

   where **p** is the relativistic momentum and **v** is the velocity of the rocket.

$$M = M_i / M_f$$

where M_i is the initial mass plus matter–antimatter and M_f is the final mass of the rocket (payload).

(V) Relativistic kinematics of constant acceleration

1. $v = at / \{(1 + (at/c)^2\}^{1/2}$

where **a** is the acceleration of the rocket as *determined in the spaceship by the force of a unit mass on a spring*; **t** is *the time as measured from starbase*, and v is *the velocity as measured from starbase.*

$$\mathbf{2.}\quad \mathbf{t' = c/a\ \text{arcsinh}\ (at/c)}$$

where **t, t'**, and **c** have been defined before, and

$$\mathbf{3.}\quad \mathbf{L = c^2/a\ \{[\,1 + (at/c)^2]^{1/2} - 1\}}$$

$$\mathbf{L = c^2/a\ (\cosh at/c\ - 1)}$$

where **L** is the distance travelled *as measured from the starbase*, and **a** and **c** have already been defined..

(VI) Relativistic addition of velocities

$$\mathbf{V_{relative} = [(v_2 - v_1) / (1 - v_1 v_2 / c^2)\,]} \quad \text{(absolute value!)}$$

where $\mathbf{V_{relative}}$ is the relative velocity between spaceship_1 and spaceship_2, as *seen from starbase.* Note that velocities are vector quantities and therefore the absolute value is taken for $\mathbf{V_{relative}}$ **.**

Appendix 2

1 An article, taken from the Internet:

IL ** http://math.ucr.edu/home/baez/physics/Relativity/SR/rocket.html**

Updated by Don Koks 2006.
Fuel numbers added by Don Koks 2004.
Updated by Phil Gibbs 1998.
Thanks to Bill Woods for correcting the fuel equation.
Original by Philip Gibbs 1996.

The following is based on the article you can download from the above IL. This article includes the relativistic rocket equations as well as the derivation of equation six below:

$$\mathbf{M = \gamma(1 + v/c) - 1 = \exp(aT/c) - 1 \ldots\ldots\ldots\ldots 6}$$

The Relativistic Rocket

The theory of relativity sets a severe limit to our ability to explore the galaxy in space ships. As an object approaches the speed of light, more and more energy is needed to accelerate it further.

LCP 14 STAR TREK

(See Fig.) To reach the speed of light an infinite amount of energy would be required. It seems that the speed of light is an absolute barrier which cannot be reached or surpassed by massive objects. Given that the galaxy is about 100,000 light years across there seems little hope for us to get very far in galactic terms unless we can overcome our own mortality.

Science fiction writers can make use of *worm holes* or *warp drives* to overcome this restriction, but it is not clear that such things can ever be made to work in reality. Another way to get around the problem may be to use the relativistic effects of time dilation and length contraction to cover large distances within a reasonable time span for those aboard a space ship. If a rocket accelerates at 1*g* (9.81 m/s^2) the crew will experience the equivalent of a gravitational field with the same strength as that on Earth (See LCP 14). If this could be maintained for long enough they would eventually receive the benefits of the relativistic effects which improve the effective rate of travel.

What then, are the appropriate equations for the relativistic rocket?

First of all we need to be clear what we mean by continuous acceleration at 1*g*. The acceleration of the rocket must be measured at any given instant in a non-accelerating frame of reference travelling at the same instantaneous speed as the rocket (see relativity FAQ on accelerating clocks). This acceleration will be denoted by *a*.

The proper time as measured by the crew of the rocket (i.e. how much they age) will be denoted by ***T*,** and the time as measured in the non-accelerating frame of reference in which they started (e.g. Earth) will be denoted by **t**.

We assume that the stars are essentially at rest in this frame.

The distance covered as measured in the non-accelerating frame of reference will be denoted by **d** and the final speed ***v*.** The time dilation or length contraction factor at any instant is the gamma factor **γ.**

The relativistic equations for a rocket with constant positive acceleration $a > 0$ are the following. First, define the hyperbolic trigonometric functions sh, ch, and th (also known as sinh, cosh, and tanh):

$$\mathrm{sh}\, x = (e^{x} - e^{-x})/2$$

$$\mathrm{ch}\, x = (e^{x} + e^{-x})/2$$

LCP 14 STAR TREK

$$\mathrm{th}\ x = \mathrm{sh}\ x/\mathrm{ch}\ x$$

$$\gamma = (1 - (v/c)^2)^{-1/2}$$

Using these, the rocket equations are

$$t = (c/a)\,\mathrm{sh}(aT/c) = [(d/c)^2 + 2d/a]^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 1$$

$$d = (c^2/a)\,[\mathrm{ch}(aT/c) - 1] = (c^2/a)\,(\,[1 + (at/c)^2]^{1/2} - 1)\ldots\ldots..\ 2$$

$$v = c\,\mathrm{th}(aT/c) = at / [1 + (at/c)^2]^{1/2}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..\ 3$$

$$T = (c/a)\,\mathrm{sh}^{-1}(at/c) = (c/a)\,\mathrm{ch}^{-1}\,[ad/c^2 + 1]\ldots\ldots\ldots\ldots\ldots\ldots.4$$

$$\gamma = \mathrm{ch}(aT/c) = [1 + (v/c)^2]^{-1/2} = [1 + (at/c)^2]^{1/2} = ad/c^2 + 1\ldots\ldots.5$$

$$M/m = \gamma(1 + v/c) - 1 = \exp(aT/c) - 1\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..6$$

t = time, as measured in the space port

T = time, as measured in the space craft

a = acceleration = 1.03 ly/y^2

M/m = Mass ratio = Initial mass over final mass

t = time, as measured in the space port

(The derivation of some of these equations will be given later)

These equations are valid in any consistent system of units such as seconds for time, metres for distance, metres per second for speeds and metres per second squared for accelerations. In these units $c = 3 \times 10^8$ m/s (approx).

However, it is easier to use units of years for time and light years for distance**.** Then
c = 1 lyr/yr and g = 1.03 lyr/yr^2.

LCP 14 STAR TREK

Using EXCELL calculations, here are some typical answers for $\boldsymbol{a} = 1\boldsymbol{g}$. (See Appendix)

Distance(d)	(lyr)	Time(t) (yr)	Velocity(v)	Time(T) (yr)	Gamma	Mass Ratio
Nearest star	4.3	5.18	0.982889536	2.30	5.429	9.765107289
	27	27.9	0.99939742	3.93	28.81	56.60263968
	100	101	0.999953771	5.18	104	206.9951922
	500	501	0.999998122	6.73	516	1030.999031
	1000	1000	0.99999953	7.41	1031	2060.999515
	10000	10000	0.999999995	9.64	10301	20600.99995
	100000	100000	1	11.88	103000.9	206001

So in theory you can travel across the galaxy in just 12 years of your own time. If you want to arrive at your destination and stop then you will have to turn your rocket around half way and decelerate at 1*g*. In that case it will take nearly twice as long in terms of proper time *T* for the longer journeys; the Earth time *t* will be only a little longer, since in both cases the rocket is spending most of its time at a speed near that of light. (We can still use the above equations to work this out, since although the acceleration is now negative, we can "run the film backwards" to reason that they still must apply.)

Here are some of the times you will age when journeying to a few well known space marks, arriving at low speed:

4.3 ly	**nearest star**	**3.6 years**
27 ly	**Vega**	**6.6 years**
30,000 ly	**Center of our galaxy**	**20 years**
2,000,000 ly	**Andromeda galaxy**	**28 years**

For distances bigger than about a thousand million light years, the formulas given here are inadequate because the universe is expanding. General Relativity would have to be used to work out those cases.

LCP 14 STAR TREK

IL *** Relativistic rocket calculater.

http://www.cora.nwra.com/~gourlay/software/Java/Voyage/

How much fuel is this?

The next chart shows the amount of fuel needed (M) for every kilogramme of payload (m=1 kg).

d	Not stopping, sailing past:	M
4.3 ly	Nearest star	10 kg
27 ly	Vega	57 kg
30,000 ly	Center of our galaxy	62 tonnes
2,000,000 ly	Andromeda galaxy	4,100 tonnes

This is a lot of fuel--and remember, we are using a motor that is 100% efficient!

What if we prefer to stop at the destination? We accelerate to the half way point at 1g and then immediately switch the direction of our rocket so that we now decelerate at 1g for the rest of second half of the trip. The calculations here are just a little more involved since the trip is now in two distinct halves (and the equations at the top assume a positive acceleration only). Even so, the answer turns out to have exactly the same form: $M/m = \exp(aT/c) - 1$, except that the proper time T is now almost twice as large as for the non-stop case, since the slowing-down rocket is losing the ageing benefits of relativistic speed. This dramatically increases the amount of fuel needed:

d	Stopping at:	M
4.3 ly	**Nearest star**	**38 kg**
27 ly	**Vega**	**886 kg**
30,000 ly	**Center of our galaxy**	**955,000 tonnes**
2,000,000 ly	**Andromeda galaxy**	**4.2 thousand million tonnes**

Compare these numbers to the previous case: they are hugely different! Why should that be?

Other fuel options

Well, this is probably all just too much fuel to contemplate. There are a limited number of solutions that don't violate energy-momentum conservation or require hypothetical entities such as tachyons or worm holes.

It may be possible to scoop up hydrogen as the rocket goes through space, using fusion to drive the rocket. This would have big benefits because the fuel would not have to be carried along from the start. Another possibility would be to push the rocket away using an Earth-bound grazer directed onto the back of the rocket. There are a few extra technical difficulties but expect NASA to start looking at the possibilities soon.

You might also consider using a large rotating black hole as a gravitational catapult but it would have to be *very* big to avoid the rocket being torn apart by tidal forces or spun at high angular velocity. If there is a black hole at the centre of the Milky Way, as some astronomers think, then perhaps if you can get that far, you can use this effect to shoot you off to the next galaxy.

One major problem you would have to solve is the need for shielding. As you approach the speed of light you will be heading into an increasingly energetic and intense bombardment of cosmic rays and other particles. After only a few years of 1*g* acceleration even the cosmic background radiation is Doppler shifted into a lethal heat bath hot enough to melt all known materials.

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IL ***
http://scienceworld.wolfram.com/physics/Tachyon.html

Graphical representation of motion based on the relativistic rocket equations, already discussed earlier on page 33:

$$t = (c/a)\,\mathrm{sh}(aT/c) = [(d/c)^2 + 2d/a]^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 1$$

$$d = (c^2/a)\,[\mathrm{ch}(aT/c) - 1] = (c^2/a)\,(\,[1 + (at/c)^2]^{1/2} - 1) \ldots\ldots\ldots \quad 2$$

$$v = c\,\mathrm{th}(aT/c) = at / [1 + (at/c)^2]^{1/2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3$$

$$T = (c/a)\,\mathrm{sh}^{-1}(at/c) = (c/a)\,\mathrm{ch}^{-1}\,[ad/c^2 + 1] \ldots\ldots\ldots\ldots\ldots\ldots 4$$

$$\gamma = \mathrm{ch}(aT/c) = [1 + (v/c)^2]^{1/2} = [1 + (at/c)^2]^{1/2} = ad/c^2 + 1 \ldots\ldots 5$$

And now we add the equation that connects mass ratio, acceleration and the time T:

$$M/m = \gamma(1 + v/c) - 1 = \exp(aT/c) - 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

(We will derive some of these equations later in the Appendix).

It is instructive to study the equations above graphically. If we change the units this way:

Distance: Light years (ly). **Velocity**: fraction of speed of light c. **Time**: years (t, T)
Aceleration: g (9.81 m/s^2). Here : c =1, a = g = 1.03 ly/ y^2, it is much easier to calculate values, especially when using a calculator.

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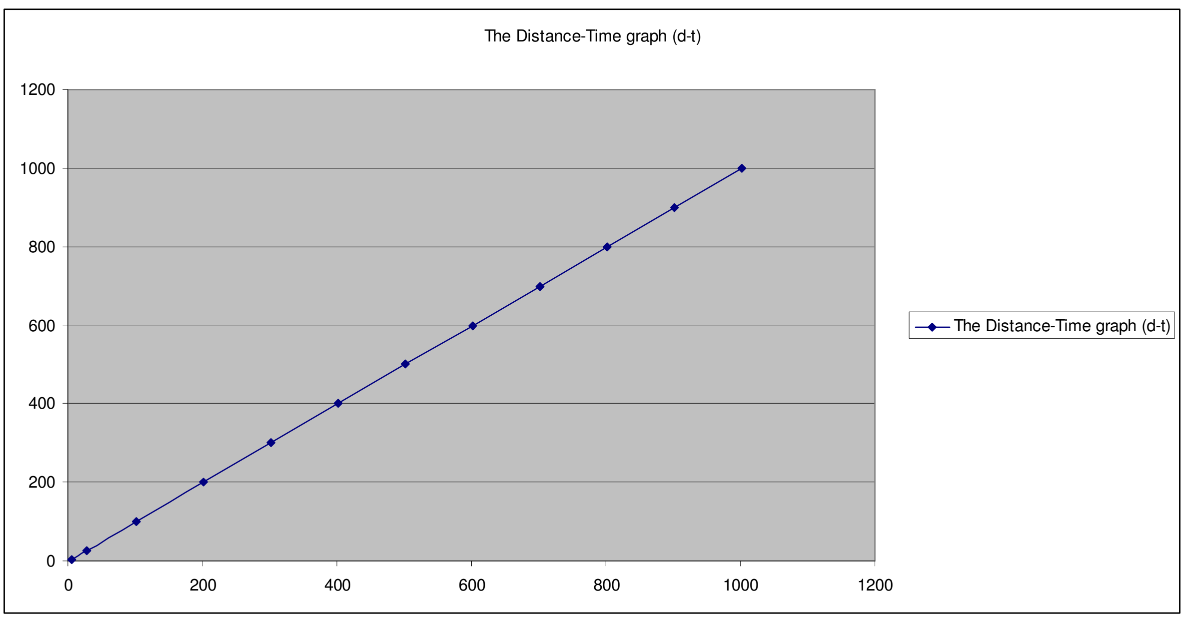
Working with EXCELL it is relatively easy to produce the following graphs:

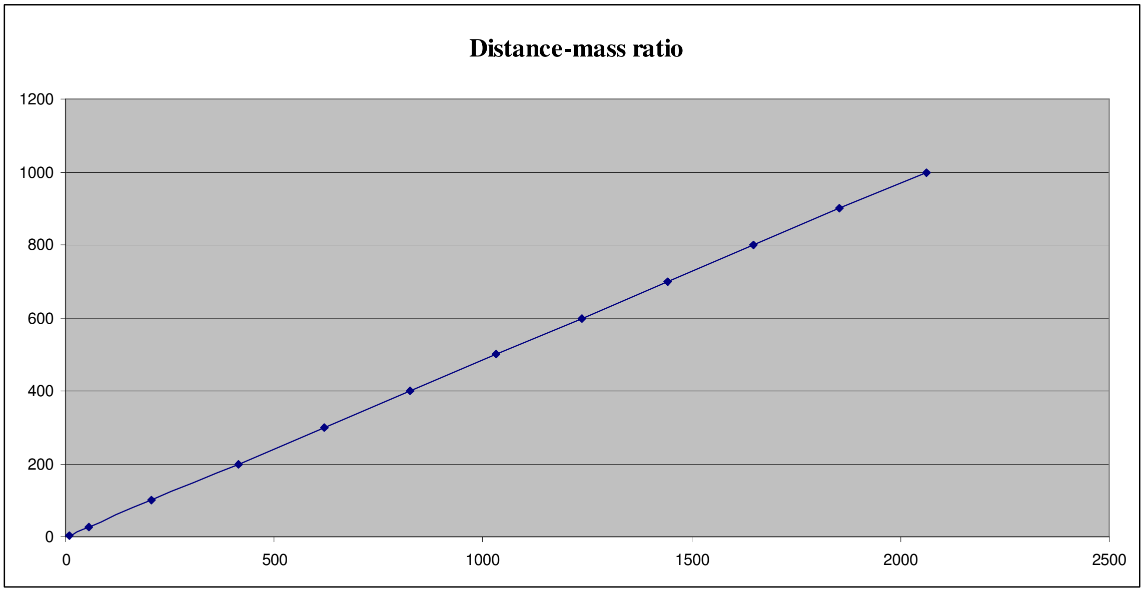
1. **The distance-time graph, as measured in the space port.**



Note that we are using $\mathbf{d = (c^2/a)\ (\ [1 + (at/c)^2]^{1/2}\ - 1)}$ **.** Clearly, distance travelled (as seen from the space port) in this equation can be written as

$\mathbf{d \approx t}$**,** or $\mathbf{D = kt}$, where **k** is the constant of proportionality that can be calculated from the slope of the graph. What are the units of this constant?

2. Distance travelled and the mass ratio $M = M_i / M_f$ (as far as 1000 ly)



Finally:

Using equation 2 : $\mathbf{d = (c^2/a)\,[ch(aT/c) - 1]}$ and equation 6:

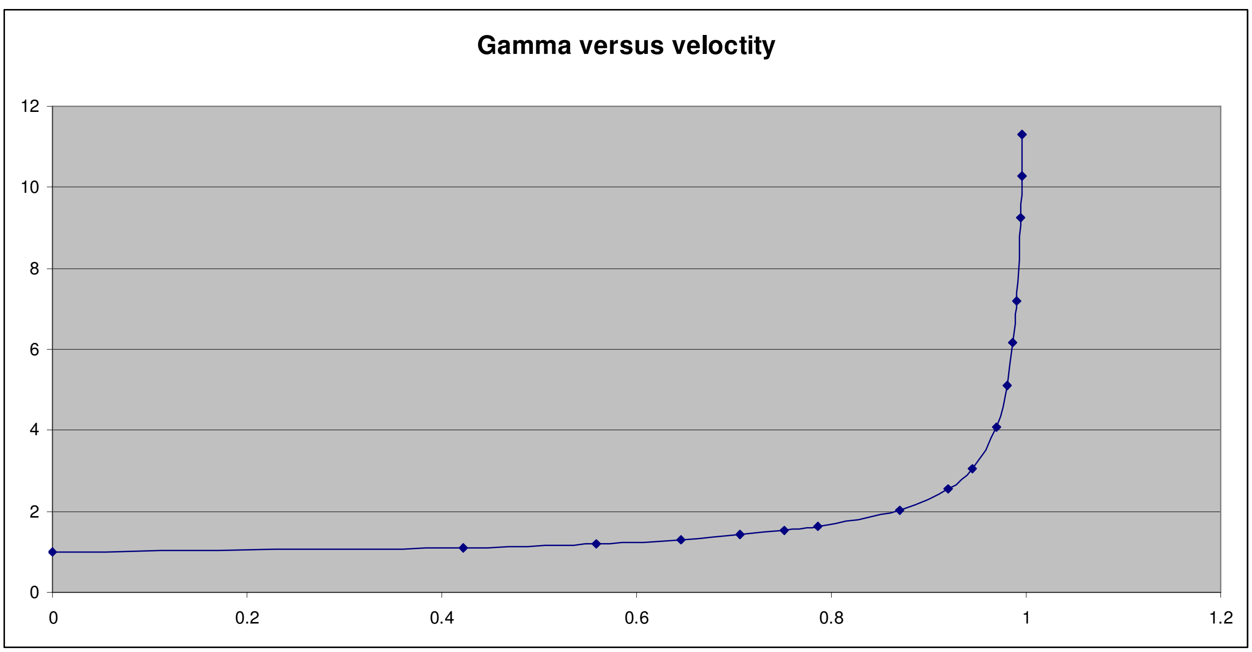
we obtain: $\mathbf{M/m = exp(aT/c) - 1,}$

it follows that $\mathbf{aT/c = ln\,(M/m)}$ and substituting into 6 we get:

LCP 14 STAR TREK

$$d = (c^2/a)\{\mathrm{ch}\ \ln(M/m) - 1\}$$

4. Gamma $(1-(v/c)^2)^{-1/2}$ as a function of velocity expressed as (v/c).



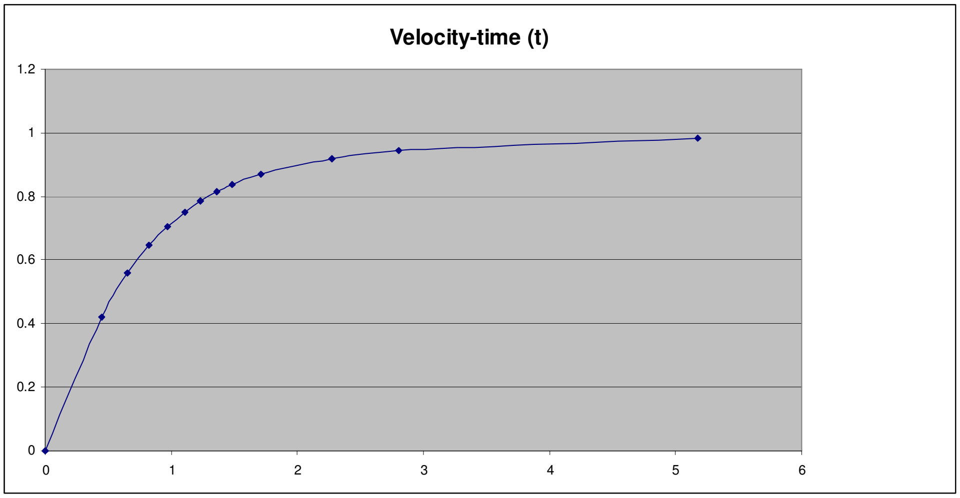
Since $\gamma = (1-(v/c)^2)^{-1/2}$ we can solve for the velocity.

Therefore: $\mathbf{v = c/\gamma\ (\gamma^2 - 1)^{1/2}}$

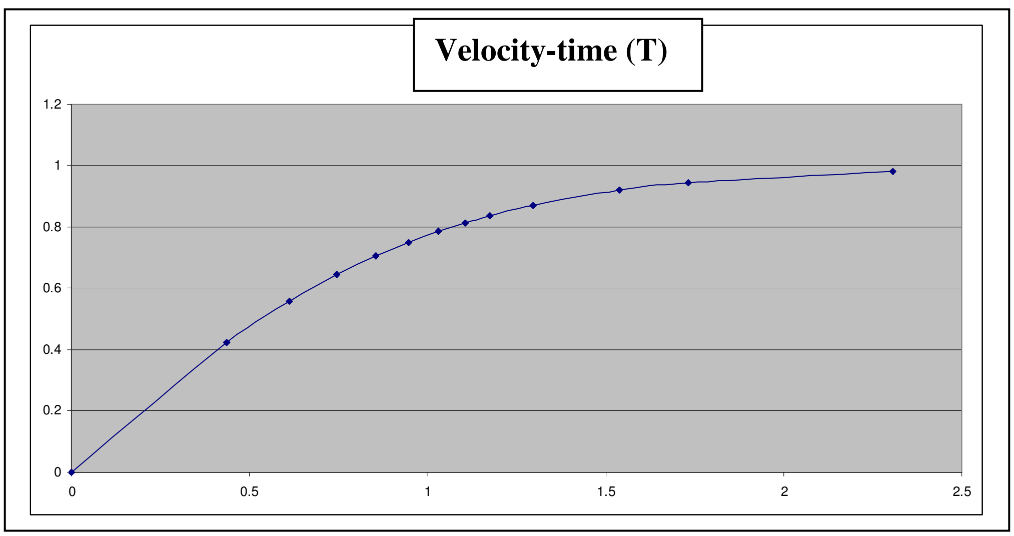
LCP 14 STAR TREK

You should now sketch a graph of

5. Velocity of the space craft, as measured from Starbase.



6. Velocity of the space craft, as measured in the space craft



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Derivation of some of the equations of relativistic rocket motion:

(Note: equation 6: **$M/m = \gamma(1 + v/c) - 1 = \exp(aT/c) - 1$………………………………..6** was derived earlier).

The derivation of most of the relativistic equations given above can be found in standard university textbooks. It is more difficult, however, to find the derivation of the equations of relativistic rocket motion (See above).

This so, because we have acceleration involved.

The derivation of $v = c\,\mathrm{th}(aT/c) = at / [1 + (at/c)^2]^{1/2}$…………………………. 3

Special relativity and accelerated frames of reference.

IL *** Short note on STR and accelerating frames of reference
http://math.ucr.edu/home/baez/physics/Relativity/SR/acceleration.html

Partly taken from IL above:

It is a common misconception that the STR cannot handle accelerating objects or accelerating reference frames. It is claimed that general relativity is required because special relativity only applies to inertial frames. This is not true. Special relativity treats accelerating frames differently from inertial frames but can still deal with them. Accelerating objects can be dealt with without even calling upon accelerating frames.

A simple problem is to solve the motion of a body which accelerates constantly. What does this mean? We don't mean that its acceleration as measured by an inertial observer is constant. We mean that it is moving so that the acceleration measured in an inertial frame travelling at the same instantaneous velocity as the object is the same at any moment.

If it was a rocket and you were on board you would experience a constant g force. This problem can be solved in a number of ways. One way to solve it is to think of the object as passing constantly from one inertial frame to another in such a way that its change of speed in a fixed time interval is seen as a Lorentz boost is always the same. From our understanding of adding velocities we can see that the..

> rapidity *r* of the object must be increasing at a constant rate *a* with respect to the proper time of the object *T*.

The acceleration **a** is related to velocity ***v*** by the equation:

$$\mathbf{v = c \tanh(aT/c)}$$

This can also be written as $$\mathbf{v = at / [1 + (at/c)^2]^{1/2}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

The relativistic rocket equation for distance:

$$\mathbf{d = (c^2/a)\,[ch(aT/c) - 1] = (c^2/a)\,(\,[1 + (at/c)^2]^{1/2} - 1)\ldots\ldots\; 2}$$

Suppose that, as viewed in the accelerating rocket (constant acceleration) the rocket is instantaneously at rest, the rocket experiences a constant acceleration a'. We know from the STR that

$$\mathbf{V_{relative} = [(v' + v) / (1 + v'\, v/ c^2)\,]} \quad \text{(absolute value!)}$$

where $\mathbf{V_{relative}}$ is the relative velocity , or the velocity as measured from Starbase.

First, it can be shown that $$\mathbf{\Delta\, v = \{1- (v/c)^2\,)\, \Delta\, v'}$$

by differentiating with respect to v'. Here **Δ v** is the change of velocity as measured on Starbase (or the Earth). We then write:

$$\mathbf{\Delta\, v / \{1- (v/c)^2\,) = (dv' / dt'\,)\, \Delta\, t' = a't'}$$

where **Δ t'** is the time interval as measured in the rocket's frame of reference. If the rocket starts at t' = 0 from rest with respect to the Starport, we integrate the left and right-hand sides to obtain:

$$\mathbf{c^2 \int_0^{v} dv / (c^2 - v^2\,) = a' \int dt'.}$$

LCP 14 STAR TREK

Using general integration tables we can show that this is:

$$c \tanh^{-1} (v/c) = a't'$$

Therefore: $$v/c = \tanh\ (a't'/c)$$

To find the total displacement L of the rocket as viewed in Starport a further integration is required:

$$L = \int v\, dt = \int v\,(dt' / (1- (v/c)^2)^{1/2}$$

It follows from that $dt' = 1/a'\ (c^2 dv / (c^2 - v^2)$

so that becomes $L= c^3 / a' \int_0^v \{v / (c^2 - v^2)^{3/2})dv = c^3 / a' \{ 1/ (c^2 - v^2)^{1/2} - 1 \}$

Using and $L= c^2 / a' \{\cosh (a't' / c -1\}$

This is our equation 2 for relativistic rocket travel, except that L here is equivalent to d in the in collection of relativistic equations given above. Note that we have used **L** rather than **d** in order not confuse the **d** (distance) with **d** (a differential notation).

We wrote this equation as $d= c^2 / a' \{\cosh (a't' / c -1\}$

Derivation of equation 6:

$$M /m = \gamma(1 + v/c) – 1 = \exp(aT/c) – 1 \ldots\ldots$$

(Partly taken from IL)

Based on conservation of energy:

The total energy before blast-off is (in the Earth frame)

$$E_{initial} = (M+m)c^2$$

At the end of the trip the fuel has all been converted to radiation with energy E_L, so the total energy is now

$$E_{final} = \gamma mc^2 + E_L$$

LCP 14 STAR TREK

By the conservation of energy principle these must be equal, so here is our first conservation equation:

$$(\mathbf{M+m})\mathbf{c}^2 = \gamma \mathbf{mc}^2 + \mathbf{E_L} \quad \text{........ (1)}$$

<u>Based on conservation of momentum</u>

The total momentum before blast-off is zero in the Earth frame.

$$\mathbf{p_{initial} = 0}$$

At the trip's end the fuel has all been converted to light with momentum of magnitude E_L/c, but in the opposite direction to the rocket. So the final momentum is

$$\mathbf{p_{final}} = \gamma\ \mathbf{mv - E_L/c}$$

By conservation of momentum these must be equal, so our second conservation equation is:

$$\mathbf{0} = \gamma\ \mathbf{mv - E_L/c} \quad \text{........ (2)}$$

Eliminating E_L from equations (1) and (2) gives

$$(\mathbf{M+m})\mathbf{c}^2 - \gamma\mathbf{mc}^2 = \gamma\mathbf{mvc}$$

so that the fuel:payload ratio is

$$\mathbf{M/m} = \gamma(\mathbf{1 + v/c}) - \mathbf{1}$$

This equation is true irrespective of how the ship accelerates to velocity v, but if it accelerates at constant rate a then

$$\mathbf{M/m} = \gamma(\mathbf{1 + v/c}) - \mathbf{1}$$

You can show that

$$\mathbf{M/m = \cosh(aT/c)[\ 1 + \tanh(aT/c)\ ] - 1}$$

or

$$\mathbf{M/m = \exp(aT/c) - 1}$$

Note: The last version is used in the graphing above.

So we can calculate the distance travelled (as seen from Starbase) in terms of the mass ratio **M/m.**

More about Tachyons

Tachyons are a putative class of particles which able to travel faster than the speed of light. Tachyons were first proposed by physicist Arnold Sommerfeld, and named by Gerald Feinberg. The word tachyon derives from the Greek ταχύς (*tachus*), meaning "speedy." Tachyons have the strange properties that, when they lose energy, they gain speed. Consequently, when tachyons gain energy, they slow down. The *slowest* speed possible for tachyons is the speed of light.

Tachyons appear to violate causality (the so-called causality problem), since they could be sent to the past under the assumption that the principle of special relativity is a true law of nature, thus generating a real unavoidable time paradox (Maiorino and Rodrigues 1999). Therefore, it seems unavoidable that if tachyons exist, the principle of special relativity must be false, and there exists a unique time order for all observers in the universe *independent* of their state of motion.

Tachyons can be assigned properties of normal matter such as spin, as well as an antiparticle (the antitachyon). And amazingly, modern presentations of tachyon theory actually allow tachyons to actually have real mass (Recami 1996).

It has been proposed that tachyons could be produced from high-energy particle collisions, and tachyon searches have been undertaken in cosmic rays. Cosmic rays hit the Earth's atmosphere with high energy (some of them with speed almost 99.99% of the speed of light) making several collisions with the molecules in the atmosphere. The particles made by this collision interact with the air, creating even more particles in a phenomenon known as a cosmic ray shower. In 1973, using a large collection of particle detectors, Philip Crough and Roger Clay identified a putative superluminal particle in an air shower, although this result has never been reproduced.

IL
http://en.wikipedia.org/wiki/Tachyon

Today, in the framework of quantum field theory, tachyons are best understood as signifying an instability of the system and treated using tachyon condensation, rather than as real faster-than-light particles, and such instabilities are described by tachyonic fields. According to the contemporary and widely accepted understanding of the concept of a particle, tachyon particles are too unstable to be treated as existing.[4] By that theory, faster than light information transmission and causality violation with tachyons are impossible on both grounds: they are non-existent in the first place (by tachyon condensation)[4] and even if they existed (by Feinberg's analysis[3]) they wouldn't be able to transmit information (also by Feinberg's analysis[3]). Despite the theoretical arguments against the existence of tachyon particles, experimental searches have been conducted to test the assumption against their existence; however, no experimental evidence for or against the existence of tachyon particles has been found.[5]

http://caribe777.blogspot.com/2007/12/faster-than-light-particles.html

The tachyon, if it existed, would have a number of fascinating properties. Unlike ordinary particles, it would have to decrease in mass as it went faster, meaning that the speed of light—at which its mass would be infinite—would be just below its slowest possible speed. Likewise, adding energy to the tachyon would slow it down, rather than speed it up; to slow it all the way down to the speed of light would require infinite energy. For a long time, physicists believed that a tachyon’s mass would have to be an imaginary number—a number with a factor that’s the square root of –1—though more recent formulations of tachyon theory suggest that such a particle could have a real mass. Most intriguingly, a tachyon, if it is to adhere to the principle of relativity, would actually be able to travel backward in time—seemingly making all sorts of trouble for the notion of causality.

IL ****

http://en.wikipedia.org/wiki/Tachyon

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LCP 14 STAR TREK